

## Universal photonic tunneling time

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(Received 9 February 2001; published 20 July 2001)

We consider photonic tunneling through evanescent regions and obtain general analytic expressions for the transit (phase) time  $\tau$  (in the opaque barrier limit) in order to study the recently proposed "universality" property according to which  $\tau$  is given by the reciprocal of the photon frequency. We consider different physical phenomena (corresponding to performed experiments) and show that such a property is only an approximation. In particular, we find that the "correction" factor is a constant term for total internal reflection and quarter-wave photonic band gap, while it is frequency dependent in the case of an undersized waveguide and distributed Bragg reflector. The comparison of our predictions with the experimental results shows quite good agreement with observations and reveals the range of applicability of the approximate "universality" property.

DOI: 10.1103/PhysRevE.64.026609

PACS number(s): 42.50.-p, 03.65.Xp

### I. INTRODUCTION

In recent times, some photonic experiments [1–6] dealing with evanescent mode propagation have drawn attention because of their intriguing results. All such experiments have measured the time required for the light to travel through a region in which only evanescent propagation occurs, according to classical Maxwell electrodynamics. If certain conditions are fulfilled (i.e., in the limit of opaque barriers), the transit times obtained are usually *shorter* than the corresponding ones for real (not evanescent) propagation through the same region. Due to the experimental setup, this has been correctly interpreted in terms of group velocities [7] greater than  $c$  inside the region considered. Although there has been some confusion in the scientific community, leading also to several different definitions of the transit time [8], these results are not at odds with Einstein causality since, according to Sommerfeld and Brillouin [9], the front velocity rather than the group velocity is relevant for this. Waves that are solutions of the Maxwell equations always travel in vacuum with a front velocity equal to  $c$  while, in certain conditions, their phase and group velocities can be different from  $c$  [10]. It is worthwhile to observe that the quoted experiments were carried out in studying different phenomena (undersized waveguide, photonic band gap, total internal reflection) and exploring different frequency ranges (from the optical to the microwave region).

The interest in such experiments is driven by the fact that evanescent mode propagation through a given region can be viewed as a photonic tunneling effect through a "potential" barrier in that region. This has been shown, for example, in Ref. [11] using the formal analogy between the (classical) Helmholtz wave equation and the (quantum mechanical) Schrödinger equation (see also Ref. [12]). In this respect, the photonic experiments are very useful in studying the ques-

tion of tunneling times, since experiments involving charged particles (e.g., electrons) are not yet sensitive enough to measure transit times due to some technical difficulties [13].

From an experimental point of view, the transit time  $\tau$  for a wave packet propagating through a given region is measured as the interval between the arrival times of the signal envelope at the two ends of that region, whose distance apart is  $D$ . In general, if the wave packet has a group velocity  $v_g$ , this means that  $\tau = D/v_g$ . Since  $v_g = d\omega/dk$  ( $k$  is the wave vector,  $\omega$  the angular frequency), then we can write [14]

$$\tau = \frac{d\phi}{d\omega}, \quad (1)$$

where  $d\phi = D dk$  is the phase difference acquired by the packet in the region considered. The above argument works for matter particles in quantum mechanics also, on changing the roles of angular frequency and wave vector into the corresponding ones of energy and momentum through the Planck–de Broglie relations.

However, difficulties arise when we deal with tunneling times, since inside a barrier region the wave vector (or the momentum) is imaginary, and hence no group velocity can be defined. As a matter of fact, different definitions of the tunneling time exist. While we refer the reader to the quoted literature [8], here we use the simple definition of phase time which coincides with Eq. (1). In fact, although  $v_g$  seems meaningless in this case, nevertheless Eq. (1) is meaningful for evanescent propagation also. The point of view adopted takes advantage of the fact that experimental results [1–6] seem to confirm the definition of phase time for the tunneling transit time.

Recently, Haibel and Nimtz [6] noted that, regardless of the different phenomena studied, all experiments have measured photonic tunneling times that are approximately equal to the reciprocal of the frequency of the radiation used in the given experiment. Such a "universal" behavior is quite remarkable in view of the fact that, although photonic barrier

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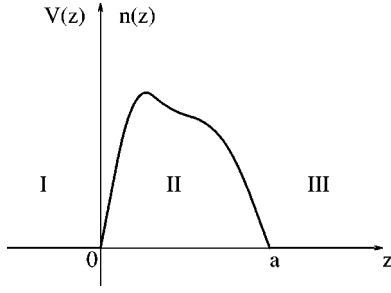


FIG. 1. A barrier potential  $V(z)$  for a particle or a barrier refractive index  $n(z)$  for an electromagnetic wave.

traversal takes place in all the quoted experiments, nevertheless the boundary conditions are peculiar to each experiment.

In the present paper we carefully study the proposed universality starting from a common feature of tunneling phenomena and, in the following section, derive a general expression for the transit (phase) time. Different experiments implement different dispersion relations for the barrier region. We then analyze each particular experiment in Secs. III, IV, and V and compare theoretical predictions with experimental observations. Finally, in Sec. VI, we discuss our results and give conclusions.

Note that, unlike other possible analyses (see, for example, the comparison with a photonic band gap experiment in [15]), we deal with only tunneling times, which have been directly observed, and not with velocities which, in the present case, are derived from transit times.

## II. PHASE TIME AND DISPERSION RELATION

In this paper we study one-dimensional (1D) problems or, in general, phenomena in which evanescent propagation takes place along one direction, say  $z$ . Let us then consider a particle or a wave packet moving along the  $z$  axis entering a region  $[0, a]$  with a potential barrier  $V(z)$  or a refractive index  $n(z)$ , as depicted in Fig. 1. The energy/frequency of the incident particle/wave is below the maximum of the potential or cutoff frequency. For all experiments will consider, the barrier can be modeled as a square one, in which  $V(z)$  or  $n(z)$  is constant in regions I, II, III but different from one region to another. We also assume that  $V(z)$  or  $n(z)$  is equal in I and III and take this value as the reference one.

The propagation of the particle/wave through the barrier is described by a scalar field  $\psi$  representing the Schrödinger wave function in the particle case or some scalar component of the electric or magnetic field in the wave case. (The precise meaning of  $\psi$  in the case of wave propagation depends on the particular phenomenon we consider. However, the aim of this paper is to show that a common background for all tunneling phenomena exists.) Given the formal analogy between the Schrödinger equation and the Helmholtz equation [11,12], this function takes the following values in regions I, II, III, respectively:

$$\psi_I = e^{ikz} + R e^{-ikz}, \quad (2)$$

$$\psi_{II} = A e^{-\chi z} + B e^{\chi z}, \quad (3)$$

$$\psi_{III} = T e^{ik(z-a)}, \quad (4)$$

where  $k$  and  $k_2 = i\chi$  are the wave vectors ( $p = \hbar k$  is the momentum) in regions I (or III) and II, respectively. Note that we have suppressed the time dependent factor  $e^{i\omega t}$ . Obviously, the physical field is represented by a wave packet with a given spectrum in  $\omega$ :

$$\psi(z, t) = \int d\omega \eta(\omega) e^{i(kz - \omega t)}, \quad (5)$$

where  $\eta(\omega)$  is the envelope function. Keeping this in mind we use, however, for the sake of simplicity, the simple expressions in Eqs. (2), (3), and (4). Furthermore, for the moment, we disregard the explicit expression for  $k$  and  $\chi$  in terms of the angular frequency  $\omega$  (or the relation between  $p$  and  $E = \hbar\omega$ ). As is well known, the coefficients  $R$ ,  $T$ ,  $A$ , and  $B$  can be calculated from the matching conditions at interfaces:

$$\psi_I(0) = \psi_{II}(0), \quad \psi_{II}(a) = \psi_{III}(a), \quad (6)$$

$$\psi'_I(0) = \psi'_{II}(0), \quad \psi'_{II}(a) = \psi'_{III}(a), \quad (7)$$

where the prime denotes differentiation with respect to  $z$ . Substituting Eqs. (2), (3), and (4) into Eqs. (6) and (7) we are then able to find  $R$ ,  $T$ ,  $A$ , and  $B$  and thus the explicit expression for the function  $\psi$ . Here we focus only on the transmission coefficient  $T$ ; its expression is as follows:

$$T = [1 - r^2 e^{-2\chi a}]^{-1} (1 - r^2) e^{-\chi a} \quad (8)$$

with

$$r = \frac{\chi + ik}{\chi - ik}. \quad (9)$$

The interesting limit is that of opaque barriers, in which  $\chi a \gg 1$ . All photonic tunneling experiments have mainly dealt with this case, in which “superluminal” propagation is predicted [16]. Taking this limit in Eq. (8) we have

$$T \approx 2 \left[ 1 - i \frac{k^2 - \chi^2}{2k\chi} \right]^{-1} e^{-\chi a}. \quad (10)$$

The quantity  $\phi$  in Eq. (1), relevant for the tunneling time, is just the phase of  $T$ :

$$\phi \approx \arctan \frac{k^2 - \chi^2}{2k\chi}. \quad (11)$$

The explicit evaluation of  $\tau$  in Eq. (1) depends, clearly, on the dispersion relations  $k = k(\omega)$  and  $\chi = \chi(\omega)$ . However, by substituting Eq. (11) into Eq. (1) we are able to write

$$\tau = 2 \left[ 1 + \left( \frac{k}{\chi} \right)^2 \right]^{-1} \frac{d}{d\omega} \frac{k}{\chi}, \quad (12)$$

showing that  $\tau$  depends only on the ratio  $k/\chi$ . We can also obtain a particularly expressive relation by introducing the quantities

$$\frac{k_1}{v_1} = k \frac{dk}{d\omega}, \quad \frac{k_2}{v_2} = -\chi \frac{d\chi}{d\omega}. \quad (13)$$

In fact, in this case we get

$$\tau = \frac{2}{\chi k} \left[ \frac{\chi^2}{k^2 + \chi^2} \frac{k_1}{v_1} + \frac{k^2}{k^2 + \chi^2} \frac{k_2}{v_2} \right]. \quad (14)$$

Note that, while  $k_1$  and  $k_2$  are the real or imaginary wave vectors in regions I (or III) and II,  $v_1$  and  $v_2$  represent the “real” or “imaginary” group velocities in the same regions. Obviously, an imaginary group velocity (which is the case for  $v_2$ ) has no physical meaning, but we stress that in the physical expression for the time  $\tau$  in Eq. (14) only the ratio  $k_2/v_2$  enters, which is a well-defined real quantity.

Equations (12) and (14) are very general ones (holding in the limit of opaque barriers): they apply to *all* tunneling phenomena. It is nevertheless clear that peculiarities of a given experiment enter into  $\tau$  only through the dispersion relations  $k = k(\omega)$  and  $\chi = \chi(\omega)$  or, better,  $k(\omega)/\chi(\omega)$ .

As an example of application of the general formula obtained, we consider here the case of tunneling of nonrelativistic electrons with mass  $m$  through a potential square barrier of height  $V_0$ . (In the next sections we then study in detail the three types of experiment already performed.) The electron energy is  $E = \hbar\omega$  (with  $E < V_0$ ) while the momenta involved in the problem are  $p = \hbar k$  and  $iq = \hbar k_2 = i\hbar\chi$ . In this case, the dispersion relations read as follows:

$$k = \sqrt{\frac{2m\omega}{\hbar}}, \quad (15)$$

$$\chi = \sqrt{\frac{2m(V_0 - \hbar\omega)}{\hbar^2}}, \quad (16)$$

and thus

$$\frac{k}{\chi} = \sqrt{\frac{\hbar\omega}{V_0 - \hbar\omega}}. \quad (17)$$

By substituting into Eq. (12) we immediately find

$$\tau = \frac{\hbar}{\sqrt{E(V_0 - E)}} = \frac{1}{\hbar} \frac{2m}{\chi k}. \quad (18)$$

### III. TOTAL INTERNAL REFLECTION

The first photonic tunneling phenomenon we consider is that of frustrated total internal reflection [17]. This is a two-dimensional process, but tunneling proceeds only in one direction. With reference to Fig. 2, a light beam impinges from a dielectric medium (typically a prism) with index  $n_1$  onto a slab with index  $n_2 < n_1$ . If the incident angle is greater than the critical value  $\theta_c = \arcsin n_2/n_1$ , most of the beam is reflected while part of it tunnels through the slab and emerges in the second dielectric medium with index  $n_1$ . Note that wave packets propagate along the  $x$  direction, while tunneling occurs in the  $z$  direction.

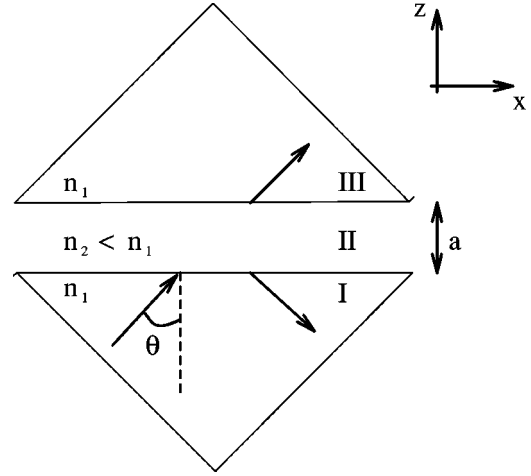


FIG. 2. Frustrated total internal reflection in a double prism.

The wave vectors  $k_1$  and  $k_2$  in regions I (or III) and II satisfy

$$k_1^2 = k_x^2 + k^2, \quad (19)$$

$$k_2^2 = k_x^2 - \chi^2, \quad (20)$$

where  $k_x$  is the  $x$  component of  $k_1$  or  $k_2$  and  $k, \chi$  are as defined in the previous section. The dispersion relations in regions I (or III) and II are, respectively,

$$k_1 = \frac{\omega}{c} n_1, \quad (21)$$

$$k_2 = \frac{\omega}{c} n_2. \quad (22)$$

These equations also define the introduced quantities

$$v_1 = \frac{c}{n_1}, \quad (23)$$

$$v_2 = \frac{c}{n_2}. \quad (24)$$

It is now very simple to obtain the tunneling time in the opaque barrier limit for this process; in fact, by substituting Eqs. (21)–(24) into Eq. (14) we find

$$\tau = \frac{1}{\omega} \frac{2k_x^2}{\chi k}. \quad (25)$$

Furthermore, using the obvious relations

$$k_x = k_1 \sin \theta = \frac{\omega}{c} n_1 \sin \theta, \quad (26)$$

$$k = k_1 \cos \theta = \frac{\omega}{c} n_1 \cos \theta, \quad (27)$$

$$\chi = \sqrt{k_1^2 \sin^2 \theta - k_2^2} = \frac{\omega}{c} \sqrt{n_1^2 \sin^2 \theta - n_2^2}, \quad (28)$$

we finally get

$$\tau = \frac{1}{\nu} \frac{n_1 \sin^2 \theta}{\pi \cos \theta \sqrt{n_1^2 \sin^2 \theta - n_2^2}}. \quad (29)$$

This formula can be directly checked in experiments. However, we first observe the interesting feature of this expression, which does satisfy the property pointed out by Haibel and Nimtz [6]. In fact, the time  $\tau$  in Eq. (29) is given, apart from a numerical factor depending on the geometry and construction of the experiment considered, by the reciprocal of the frequency of the radiation used. In a certain sense, the numerical factor can be regarded as a “correction” factor to the “universality” property of Haibel and Nimtz.

Several experiments measuring the tunneling time in the process considered have been performed [3]. In the experiment carried out by Balcou and Dutriaux [3], two fused silica prisms with  $n_1 = 1.403$  and an air gap ( $n_2 = 1$ ) were used. They employed a Gaussian laser beam of wavelength  $3.39 \mu\text{m}$  with an incident angle  $\theta = 45.5^\circ$ . Using these values in Eq. (29) we predict a tunneling time of 36.8 fs, to be compared with the experimental result of about 40 fs. As we can see, the agreement is good and the “correction” factor in Eq. (29) is quite important for this to occur (compare with the Haibel and Nimtz prediction of 11.3 fs).

In the measurements by Mugnai, Ranfagni, and Ronchi [3], the microwave region is explored, with a signal whose frequency is in the range 9–10 GHz. They used two paraffin prisms ( $n_1 = 1.49$ ) with an air gap ( $n_2 = 1$ ), while the incidence angle was about  $60^\circ$ . For this experiment we predict a tunneling time of 87.2 ps, while the experimental result is  $87 \pm 7$  ps.<sup>1</sup>

Finally, we consider the recent experiment performed by Haibel and Nimtz [6] with microwave radiation at  $\nu = 8.45$  GHz and two Perspex prisms ( $n_1 = 1.605$ ) separated by an air gap ( $n_2 = 1$ ). For an incident angle of  $45^\circ$ , from Eq. (29) we predict  $\tau = 80.8$  fs. The observed experimental result is, instead,  $117 \pm 10$  fs. In this case, the agreement is not very good (dropping the “correction” factor, Haibel and Nimtz find better agreement); probably this is due to the fact that the condition of an opaque barrier is not completely fulfilled.

#### IV. UNDERSIZED WAVEGUIDE

Let us now consider propagation through undersized rectangular waveguides as observed in [1]. In this case also, evanescent propagation proceeds along one direction (say  $z$ ) and the results obtained in Sec. II may apply. With reference

<sup>1</sup>Note that the value of 134 ps used by Haibel and Nimtz refers to the gap filled with paraffin. In this case no tunneling effect is present. We observe that for this experiment also the “correction” factor in Eq. (29) plays a crucial role for the tunneling times.

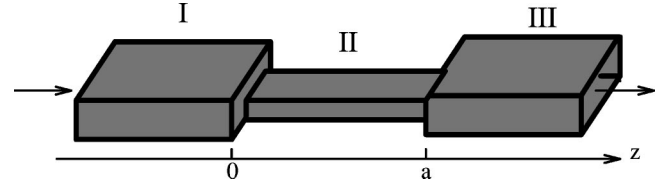


FIG. 3. A waveguide with an undersized region.

to Fig. 3, a signal propagating inside a “large” waveguide at a certain point goes through a “smaller” waveguide for a given distance  $a$ . As is well known [18], signal propagation inside a waveguide is allowed only for frequencies higher than a typical value (cutoff frequency) depending on the geometry of the waveguide. In the considered setup, the two differently sized waveguides I (or III) and II thus have different cutoff frequencies (the first one  $\omega_1$  is smaller than the second one  $\omega_2$ ), and we consider the propagation of a signal whose frequency (or range of frequencies) is larger than  $\omega_1$  but smaller than  $\omega_2$ :  $\omega_1 < \omega < \omega_2$ . In such a case, in the region  $0 < z < a$  only evanescent propagation is allowed and thus the undersized waveguide acts as a barrier for the photonic signal. With the same notation as in Sec. II, the dispersion relations in the large and small waveguides are, respectively,

$$ck = \sqrt{\omega^2 - \omega_1^2}, \quad (30)$$

$$c\chi = \sqrt{\omega_2^2 - \omega^2}, \quad (31)$$

so that

$$\frac{k}{\chi} = \sqrt{\frac{\omega^2 - \omega_1^2}{\omega_2^2 - \omega^2}}. \quad (32)$$

By substituting this expression into Eq. (12), we immediately find the tunneling time in the regime of an opaque barrier ( $\chi a \gg 1$ ):

$$\tau = \frac{1}{\nu} \frac{1}{\pi} \sqrt{\frac{\nu^4}{(\nu^2 - \nu_1^2)(\nu_2^2 - \nu^2)}}. \quad (33)$$

In contrast to what happens for tunneling in total internal reflection setups, the coefficient of the term  $1/\nu$  is not constant but itself depends on frequency. Thus, in the case of undersized waveguides, the assumed “universality” property of Haibel and Nimtz cannot apply in general; depending on the cutoff frequencies, it is only a partially approximate property for frequencies far way from the cutoff values (i.e., when the term under the square root does not strongly depend on  $\nu$ ).

Let us now compare the prediction (33) with the experimental results obtained in [1]. In the performed experiment we have microwave radiation along waveguides whose cutoff frequencies are  $\nu_1 = 6.56$  GHz and  $\nu_2 = 9.49$  GHz, respectively. The radiation frequencies are around  $\nu = 8.7$  GHz, so that tunneling phenomena occur in the undersized waveguide. By substituting these values into Eq. (33),



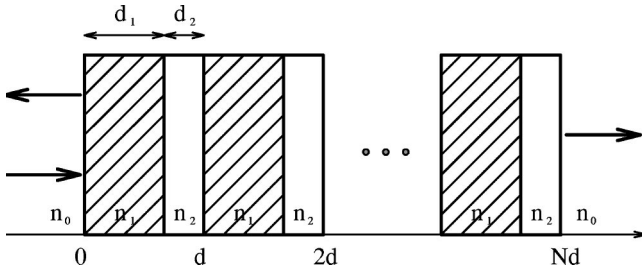


FIG. 4. An ideal photonic band gap device.

we predict a tunneling time of 128 ps, compared with the observed time of about 130 ps.

It is evident that for an undersized waveguide setup also the theory matches quite well with experiments. Note that, despite the rich frequency dependence in Eq. (33), the Haibel and Nimtz property also works quite well (although some correction is needed), since the central frequency value of the radiation used in the experiment is far enough from the cut-off values.

## V. PHOTONIC BAND GAP

The last phenomenon we consider is that of light propagation through photonic band gap materials. The ideal setup is depicted in Fig. 4. Light impinges on a succession of thin plane-parallel films composed of  $N$  two-layer unit cells of thicknesses  $d_1, d_2$  and constant, real refractive indices  $n_1, n_2$ , embedded in a medium of index  $n_0$ . It is known [19] that such a multilayer dielectric mirror possesses a (one-dimensional) ‘‘photonic band gap,’’ that is, a range of frequencies corresponding to purely imaginary values of the wave vector. In practice, it is the optical analog of crystalline solids possessing band gaps. Increasing the number of periods will result in an exponential increase of the reflectivity, and thus the opaque barrier condition can be fulfilled. In general, the study of electromagnetic properties of such materials is very complicated, and the dispersion relation we need to evaluate the phase time in the proposed formalism is quite involved for physical situations. This study was performed analytically in [15] where the dispersion relation (and other useful quantities) was derived starting from the complex transmission coefficient of the barrier considered. It is then quite meaningless to get the tunneling time from the dispersion relation obtained from the transmission coefficient, while it is easier to obtain the phase time  $\tau$  from Eq. (1) directly, where  $\phi$  is the phase of the complex transmission coefficient.

### A. Quarter-wave stack

We first consider the relevant case in which each layer is designed so that the optical path is exactly 1/4 of some reference wavelength  $\lambda_0$ :  $n_1 d_1 = n_2 d_2 = \lambda_0/4$ . In such a case,  $\lambda_0$  corresponds to the midgap frequency  $\omega_0$  ( $\lambda_0 = 2\pi c/\omega_0$ ). This condition is fulfilled in the experiments considered [2]. Finally, we further assume normal incidence of the light on the photonic band gap material.

From [15] we then obtain the following expression for the transmission coefficient:

$$T = [(AC - B) + iAD]^{-1}, \quad (34)$$

where  $A, B, C, D$  are real quantities given by

$$A = \frac{\sin N\beta}{\sin \beta}, \quad (35)$$

$$B = \frac{\sin(N-1)\beta}{\sin \beta}, \quad (36)$$

$$C = a \cos \frac{\pi\omega}{\omega_0} + b, \quad (37)$$

$$D = c \sin \frac{\pi\omega}{\omega_0}, \quad (38)$$

$$a = \frac{1 - r_{02}^2}{t_{02}t_{21}t_{12}}, \quad (39)$$

$$b = \frac{r_{12}^2(r_{02}^2 - 1)}{t_{02}t_{21}t_{12}}, \quad (40)$$

$$c = \frac{2r_{02}r_{12} - r_{02}^2 - 1}{t_{02}t_{21}t_{12}}, \quad (41)$$

$$r_{ij} = \frac{n_i - n_j}{n_i + n_j}, \quad (42)$$

$$t_{ij} = \frac{2n_j}{n_i + n_j}, \quad (43)$$

$$\sin \beta = \frac{1}{t_{12}t_{21}} \sqrt{2r_{12}^2 \left( \cos \frac{\pi\omega}{\omega_0} - 1 \right) + \sin^2 \frac{\pi\omega}{\omega_0}} \quad (44)$$

( $i, j = 1, 2$ ). The phase  $\phi$  of the transmission coefficient thus satisfies

$$\tan \phi = \frac{AD}{B - AC}. \quad (45)$$

By substituting into Eq. (1), we finally get an analytical expression for the tunneling time of light with frequency  $\nu$  close to the midgap value  $\nu_0$  for  $N$  layers:

$$\tau = \frac{1}{\nu_0} \frac{1}{2} \frac{c \sinh N\theta}{\sinh(N-1)\theta + (b-a)\sinh N\theta}, \quad (46)$$

where  $\theta$  is simply obtained from

$$\sinh \theta = \frac{1}{2} \left( \frac{n_2}{n_1} - \frac{n_1}{n_2} \right). \quad (47)$$

Note that, although the tunneling behavior is quite different if the number of periods  $N$  is an even or odd number (see, for

example, [20]), the expression for the tunneling time given in Eq. (46) [and also in Eq. (48)] is the same in both cases.

For future reference, we also report the appropriate formula for  $N=k+(1/2)$  (integer  $k$ ) multilayer dielectric mirrors. In practice, this models the case of a stratified medium whose structure has the form  $n_1 n_2 n_1 n_2 \dots n_1 n_2 n_1$  (note, however, that this is an approximation since, in general,  $d/2$  is not equal to  $a$ ). In such a case, Eq. (46) is just replaced by

$$\tau = \frac{1}{v_0} \frac{1}{2} \frac{c \cosh N\theta}{\cosh(N-1)\theta + (b-a)\cosh N\theta}. \quad (48)$$

Let us observe that, similarly to total internal reflection, at midgap the time  $\tau$  in Eq. (46) or (48) is again given by the reciprocal of the frequency times a constant ‘‘correction’’ factor.

We now analyze experimental results [2] in the light of our theoretical speculations. In the experiment performed by Steinberg, Kwiat, and Chiao, the authors used a quarter-wave multilayer dielectric mirror with a  $(HL)^5H$  structure with a total thickness of  $d=1.1 \mu\text{m}$  attached on one side of a substrate and immersed in air. Here,  $H$  represents a titanium oxide film with  $n_1=2.22$ , while  $L$  is a fused silica layer with  $n_2=1.41$ . Thus, we have approximately  $N=5+(1/2)$ . As incident light, they employed a wave packet centred at a wavelength  $\lambda_0=702 \text{ nm}$ , corresponding to the midgap frequency  $\nu_0$  of about 427 THz. By substituting these numbers in our formula (48) we predict a tunneling time  $\tau=2.66 \text{ fs}$ , corresponding to a delay time  $\Delta t$  with respect to nontunneling photons propagating at the speed of light for the distance  $d$  of  $-1.01 \text{ fs}$ . This has to be compared with the experimental result of  $\Delta t=-1.47\pm 0.2 \text{ fs}$ . However, we point out that our analytical prediction is affected by two major approximations. The first one is, as already remarked, that the experimental sample is not really a  $5+(1/2)$  periodic structure. A better approximation is achieved by using Eq. (46) with  $N=6$  and subtracting the time required for traveling at the speed of light the quarter-wave thickness  $d_2=\lambda_0/4n_2$ . In this case we have  $\tau=2.02 \text{ fs}$  or a delay time  $\Delta t=-1.65 \text{ fs}$ , which is in better agreement with the experimental result. Furthermore, in our analysis [leading to Eq. (46) or Eq. (48)] there is no room for considering an asymmetric structure (like the substrate-air one) in which the photonic band gap material is embedded. This cannot be taken into account in an analytical framework, but has to be studied using the numerical matrix transfer method, which would give quite good agreement with observations [15].

Finally, we consider the experiment carried out by Spielmann *et al.* [2] on alternating quarter-wave layers of fused silica  $L$  and titanium dioxide  $H$  having the structure of (substrate) $(HL)^N$ (air) with  $N=3,5,7,9,11$ . They used optical pulses of frequency 375 THz corresponding to the midgap frequency of their photonic band gap material. Obviously, by increasing  $N$  we have a better realization of the opaque barrier condition. From Eq. (46) with  $N=11$  [note, however, that for  $N\geq 5$  the factor  $\sinh(N-1)\theta/\sinh N\theta$  is almost constant] we have a tunneling time of 2.98 fs to be compared with the observed value of about 2.71 fs. We address the fact that, apart from the presence of the asymmetric substrate-air

structure, which introduces some approximation as discussed above, in the experiment considered the incidence of the light on the sample is not normal, the angle between the axis of the sample and the beam propagation direction being  $\approx 20^\circ$ . In this case, the described computations are only approximate ones and, again, the exact result can be obtained only through numerical implementation. Nevertheless, also within the limits of our calculations, the agreement between theory and experiment is quite good.

A final comment regards the predictions of the ‘‘universality’’ property proposed by Haibel and Nimtz. Neglecting the ‘‘correction’’ factor in Eq. (46) would yield the values of  $\Delta t=-1.33 \text{ fs}$  and  $\tau=2.67 \text{ fs}$  for the delay time in the Steinberg, Kwiat, and Chiao experiment and the transit time for the Spielmann *et al.* experiment, respectively. In both cases, the agreement with the observed values seems better than our approximate predictions, showing that the presence of an asymmetric substrate-air structure (and the non-normal incidence in the second experiment) pushes up the ‘‘correction’’ factor in Eq. (46).

## B. Distributed Bragg reflector

We now relax the assumption of a quarter-wave stack  $n_1 d_1 = n_2 d_2 = \lambda_0/4$  but, for simplicity, we consider only the case in which the photonic band gap structure is embedded into a material whose refractive index  $n_0$  is equal to that of one of the two layers of the periodic structure, that is,  $n_0 = n_2$ . We again assume normal incidence of the light on the photonic band gap material. In this case the transmission coefficient  $T$  and its phase  $\phi$  have the expressions as in Eqs. (34) and (45), where  $A, B$  are given by Eqs. (35), (36), and [15]:

$$C = a \cos \pi \Omega_+ \omega - b \cos \pi \Omega_- \omega, \quad (49)$$

$$D = -a \sin \pi \Omega_+ \omega + b \sin \pi \Omega_- \omega, \quad (50)$$

$$\Omega_{\pm} = \frac{n_1 d_1 \pm n_2 d_2}{c}, \quad (51)$$

$$\sin \beta = \frac{1}{t_{12} t_{21}} \sqrt{P + Q + R}, \quad (52)$$

$$P = r_{12}^4 \sin^2 \pi \Omega_- \omega, \quad (53)$$

$$Q = 2r_{12}^2 (\cos \pi \Omega_+ \omega \cos \pi \Omega_- \omega - 1), \quad (54)$$

$$R = \sin^2 \pi \Omega_+ \omega. \quad (55)$$

By substituting into Eq. (1) we obtain the tunneling time relative to an  $N$ -layer structure:

$$\tau = \frac{1}{v} \frac{X - Y}{Z}, \quad (56)$$

$$X = F \sin^2 \beta \cos N\beta \sin N\beta, \quad (57)$$

$$Y = G (\cos \beta \cos N\beta \sin N\beta - N \sin \beta), \quad (58)$$

TABLE I. Comparison between predicted and observed tunneling times for several experiments (FTIR, UWG, and PBG stand for frustrated total internal reflection, undersized waveguide, and photonic band gap, respectively).  $\tau_{\text{expt}}$  is the experimental result while  $\tau_{\text{th}}$  is our prediction from Eqs. (29), (33), and (46) or (56). For reference to the Haibel and Nimtz property, we also report the value  $1/\nu$ .

Phenomenon	Experiment	$1/\nu$	$\tau_{\text{th}}$	$\tau_{\text{expt}}$
FTIR	Balcou and Dutriaux [3]	11.3 fs	36.8 fs	$\sim 40$ fs
FTIR	Mugnai <i>et al.</i> [3]	100 ps	87.2 ps	$87 \pm 7$ fs
FTIR	Haibel and Nimtz [6]	120 ps	81 ps	$117 \pm 10$ fs
UWG	Enders and Nimtz [1]	115 ps	128 ps	$\sim 130$ fs
PBG( $\lambda_0/4$ )	Steinberg <i>et al.</i> [2]	2.34 fs	2.02 fs	$2.20 \pm 0.2$ fs
PBG( $\lambda_0/4$ )	Spielmann <i>et al.</i> [2]	2.67 fs	2.98 fs	$\sim 2.71$ fs
PBG	Mojahedi <i>et al.</i> [5]	103 ps	320 ps	$318 \pm 20$ ps

$$Z = 2 \sin \beta (D^2 \sin^2 N \beta + \sin^2 \beta \cos^2 N \beta), \quad (59)$$

$$F = a \Omega_+ \omega \cos \pi \Omega_+ \omega - b \Omega_- \omega \cos \pi \Omega_- \omega, \quad (60)$$

$$G = a^2 \Omega_+ \omega \sin^2 \pi \Omega_+ \omega + b^2 \Omega_- \omega \sin^2 \pi \Omega_- \omega - 2ab(\Omega_+ + \Omega_-) \omega \sin \pi \Omega_+ \omega \sin^2 \pi \Omega_- \omega. \quad (61)$$

Note that, again, the formula above for  $\tau$  holds both for even  $N$  and for odd  $N$ .

The expression obtained for the tunneling time can be directly tested by analyzing the experiment carried out by Mojahedi *et al.* [5]. In this experiment the authors used a (1D) photonic crystal composed of five polycarbonate sheets with refractive index  $n_1 = 1.66$  and thickness  $d_1 = 1.27$  cm separated by regions of air  $n_2 = 1$  with thickness  $d_2 = 4.1$  cm. The band gap was tuned to the main frequency component ( $\nu = 9.68$  GHz) of the incident microwave pulse. By measuring both the signal traveling through the photonic band gap structure and the one propagating in free space, the authors found that the pulse undergoing tunneling has a delay time  $\Delta t = -440 \pm 20$  ps with respect to the other signal. By using Eq. (56) with the above numbers we predict a tunneling time of 320 ps,<sup>2</sup> corresponding to a delay time of  $\Delta t = -438$  ps, which is in excellent agreement with the reported experimental result.

We point out that, in this case, the simple  $1/\nu$  law proposed by Haibel and Nimtz does not work, since it would predict a tunneling time  $\tau = 103$  ps or  $\Delta t = -655$  ps. This can be easily explained by looking at Eq. (56). In fact, we immediately recognize that the ‘‘correction’’ factor in this equation is strongly frequency dependent and, for the frequency of the light used in the experiment considered, it is considerably bigger than 1.

## VI. CONCLUSIONS

In this paper we have scrutinized the recently proposed [6] ‘‘universality’’ property of the photonic tunneling time, according to which the barrier traversal time for photons

propagating through an evanescent region is approximately given by the reciprocal of the photonic frequency, irrespective of the particular setup employed. To this end, the transit time in the relevant region, defined here as in Eq. (1), needs to be computed for the different phenomena explored, and in Sec. II we have given general expressions for this time in the opaque barrier limit. The peculiarities of a given photonic setup enter into these expression only through the dispersion relation relating the wave vector and the frequency. In more detail, we have shown how knowledge of the ratio between the wave vectors in the barrier region and outside it, as a function of the photon frequency, is sufficient to evaluate the transit time  $\tau$  in Eq. (12).

Several specific cases, corresponding to the different classes of experimentally investigated phenomena, were then considered. In particular, in Sec. III we studied light propagation in a setup in which the evanescent region is provided by total internal reflection, while in Sec. IV propagation through undersized waveguides was considered; and, finally, in Sec. V the case of a photonic band gap was analyzed. The relevant results for the three mentioned phenomena are given in Eqs. (29), (33), and (46) or (56), respectively. As can easily be seen from these expressions, the frequency dependence of the tunneling time for the cases of total internal reflection and quarter-wave photonic band gap is just as predicted by the property outlined by Haibel and Nimtz [6], although we have derived a ‘‘correction’’ factor depending on the geometry and on the intrinsic properties of the sample (this factor is not far from unity). On the contrary, such a factor is frequency dependent for undersized waveguides and distributed Bragg reflectors, revealing a richer dependence of  $\tau$  on  $\nu$  than the simple  $1/\nu$  one [see Eq. (33)]. We can then conclude that the ‘‘universality’’ property of Haibel and Nimtz is only an approximation, but it gives the right order of magnitude for the tunneling time. This conclusion holds also for undersized waveguide propagation, provided that the photon frequency is far enough from the cutoff frequencies. We then calculated the tunneling times for the different existing experiments and compared the theoretical values with the observed ones. Results are summarized in Table I, where we also report the Haibel and Nimtz prediction  $1/\nu$ . From these we can see that, in general, the agreement of our prediction with the experimental values is satisfactory. As

<sup>2</sup>Such a result was also obtained in [5] using a formalism described in [4] that is different from the one proposed here.

pointed out in the previous section, the calculations performed here for photonic band gap materials assume some approximations in treating the complex sample, which are nevertheless required to obtain analytical expressions. Our predictions suffer from this and, in the case in which the setup is designed to satisfy the quarter-wave condition  $n_1 d_1 = n_2 d_2 = \lambda_0/4$ , the simple  $1/\nu$  rule fits better with experiments, while, for general photonic band gap structures, the tunneling time displays a very complicated dependence on frequency. In this last case, as well as in all other non-

photonic band gap experiments, the “correction” factor introduced in this paper is quite relevant for the agreement with observations to be good.

#### ACKNOWLEDGMENTS

The author is indebted to Professor E. Recami for many fruitful discussions and useful information about the subject of this paper. He also gratefully acknowledges Professor A. Della Selva for discussions.

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